MAT 2379 - Spring 2011
Assignment 4 : Solutions

5.3 (3 points) This question deals with the binomial distribution with $n = 5, p = 0.39$. From Table 3.7, $P(Y = y) = \binom{n}{y} p^y (1-p)^{n-y}$, $y = 0, 1, ..., n$

a) (i) $P(Y = 0)$ =0.08
(ii) $P(Y = 1)$ =0.27
(iii) $P(Y = 2)$ =0.35
(iv) $P(Y = 3)$ =0.22
(v) $P(Y = 4)$ =0.07
(vi) $P(Y = 5)$ =0.01

5.15 (4 points) Y has a normal with mean $\mu = 176$, $\sigma = 30$
(a)

$$\begin{aligned} P(166 \leq Y \leq 186) = &\quad P\left(\frac{166-176}{30} \leq Z \leq \frac{186-176}{30}\right) \\ = &\quad P(-0.33 \leq Z \leq 0.33) \\ = &\quad 0.6293 - 0.3707 = 0.2586 \end{aligned}$$

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(b) Here we are concerned with the sampling distribution of $\overline{Y}_9$ *which* is normal with mean 176 and standard deviation $\sigma_{\overline{Y}} = \frac{\sigma}{\sqrt{9}} = 30/3 = 10$

$$\begin{aligned} P(166 \leq \overline{Y} \leq 186) = &\quad P\left(\frac{166-176}{10} \leq Z \leq \frac{186-176}{10}\right) \\ = &\quad P(-1 \leq Z \leq 1) = 0.68 \end{aligned}$$

(c) This question is just a rephrasing of the question in part (b) and hence the probability is 0.68

5.22(2 points) *Here* $\overline{Y}_n$ has a normal distribution with mean 50 and $\sigma_{\overline{Y}} = \frac{\sigma}{\sqrt{n}} = \frac{9}{\sqrt{n}}$. An area of 0.68 corresponds to ± 1 deviation on the Z-scale. Hence,
$z = 1 = \frac{51.10-50}{\frac{9}{\sqrt{n}}} = \frac{1.1\sqrt{n}}{9}$. Hence $n = 36$.

5.42 (2 points)Y has a normal with mean $\mu = 88$, $\sigma = 7$.
Here we are concerned with the sampling distribution of $\overline{Y}_5$ *which* is normal with mean 88 and standard deviation $\sigma_{\overline{Y}} = \frac{7}{\sqrt{5}} = 3.13$

$P(\overline{Y}_5 \geq 90) = P\left(Z \geq \frac{90-88}{3.13}\right) = P\left(Z \geq 0.64\right) = 1 - 0.7389 = 0.261\,1$

5.44 (6 points)Y has a normal with mean $\mu = 69.7$, $\sigma = 2.8$

(a) $P(Y \geq 72) = P\left(Z \geq \frac{72-69.7}{2.8}\right) = P\left(Z \geq 0.82\right) = 1 - 0.8939 = 0.106\,1$
(b) (i) Using the binomial distribution,
$P(both\ are > 72) = 0.2061^2 = 0.0425$
(ii) $n = 2; \sigma_{\overline{Y}} = \frac{2.8}{\sqrt{2}} = 1.979\,9$
$z = \frac{72-69.7}{1.98} = 1.161\,6$
$P(\overline{Y}_2 \geq 72) = 1 - 0.877 = 0.123$

5.47 (4 points) We are concerned with the distribution of $\hat{p}$ which is determined from the binomial with $n = 20, p = 0.2$

a) $P(\hat{p} = p) = P(X = 4) = {}_{20}C_4\ p^4 \left(1 - p\right)^{16} = 4845 \left(.2\right)^4 \left(.8\right)^{16} = 0.218\,2$
b) The event $p - 0.05 \leq \hat{p} \leq p + 0.05$ is equivalent to $0.15 \leq \hat{p} \leq 0.25$
That is , $3 \leq X \leq 5$.
We calculate using the binomial distribution

$$\begin{aligned} & P\left(X = 3\right) + P\left(X = 4\right) + P\left(X = 5\right) \\ = {} & 0.20536 + 0.21820 + 0.17456 \\ = {} & 0.5981 \end{aligned}$$

Total= 21 points